

Part B - Syllabus

MATHEMATICS (MAT)

Real Analysis: Real number system and completeness; sequences: convergence, Cauchy sequences, monotone sequences; series: absolute and conditional convergence, ratio, root, and comparison tests; continuity: uniform continuity, Lipschitz condition; differentiability: mean value theorems, Taylor's theorem; Riemann integration: Darboux sums, integrability conditions, fundamental theorem of calculus; sequences and series of functions: uniform convergence, Weierstrass M-test; improper integrals; metric spaces: open and closed sets, compactness, connectedness.

Complex Analysis: Complex number system; analytic functions; Cauchy-Riemann equations; harmonic functions; elementary functions; complex integration: Cauchy's theorem and Cauchy's integral formula; Taylor and Laurent series; singularities: removable, poles, and essential; residue theorem and its applications; evaluation of real integrals; conformal mappings; bilinear (Möbius) transformations; Rouché's theorem and argument principle.

Linear Algebra: Vector spaces: subspaces, basis, dimension, and linear independence; linear transformations: kernel, range, rank-nullity theorem; matrix algebra: rank, row reduction, system of equations; determinants: properties and Cramer's rule; eigenvalues and eigenvectors: diagonalisation, Cayley-Hamilton theorem; inner product spaces: Gram-Schmidt orthogonalisation; quadratic forms; Jordan canonical form; singular value decomposition (SVD); applications to least squares problems.

Abstract Algebra: Groups: subgroups, cosets, Lagrange's theorem, normal subgroups, quotient groups, homomorphisms, isomorphism theorems; cyclic groups and permutation groups; rings: ideals, quotient rings, ring homomorphisms; integral domains and fields; polynomial rings; principal ideal domains; unique factorisation domains; field extensions; Galois theory: Galois group, fundamental theorem of Galois theory; finite fields.

Topology: Topological spaces: open and closed sets, basis, sub-basis; continuity and homeomorphisms; subspace, product, and quotient topologies; compactness: finite intersection property, Tychonoff's theorem; connectedness and path-connectedness; separation axioms: T1, T2 (Hausdorff), T3, T4; metric topology; complete metric spaces; Baire category theorem; Urysohn's lemma; Tietze extension theorem; homotopy and fundamental group (introduction).

Ordinary Differential Equations: First-order ODEs: separable, linear, exact, Bernoulli, and homogeneous equations; higher-order linear ODEs: homogeneous and non-homogeneous; methods of undetermined coefficients and variation of parameters; Euler-Cauchy equations; systems of ODEs; power series solutions; Frobenius method; Bessel's and Legendre's equations; Sturm-Liouville problems; existence and uniqueness theorems (Picard-Lindelöf); qualitative theory: phase plane and stability.

Partial Differential Equations: Classification of second-order PDEs: elliptic, parabolic, and hyperbolic; method of characteristics; wave equation: D'Alembert's solution, separation of variables; heat equation: solution by separation of variables and Fourier series; Laplace equation: boundary value problems, Dirichlet and Neumann problems; Green's functions; Fourier transform methods; Sturm-Liouville theory; distributions and weak solutions.

Functional Analysis: Normed linear spaces: Banach spaces; bounded linear operators; dual spaces and Hahn-Banach theorem; Hilbert spaces: orthonormal bases, Riesz representation theorem; Bessel's inequality and Parseval's identity; closed graph theorem; open mapping theorem; uniform

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boundedness principle; compact operators; spectral theory: spectrum and resolvent; Fredholm theory; applications to integral equations.

Numerical Analysis: Error analysis: round-off, truncation, and propagation; interpolation: Lagrange, Newton's forward and backward, and Hermite; numerical differentiation and integration: trapezoidal, Simpson's, Gaussian quadrature; root finding: bisection, Newton-Raphson, secant method; solution of linear systems: Gauss elimination, LU decomposition, iterative methods (Jacobi, Gauss-Seidel); eigenvalue computation: power method; numerical ODEs: Euler, Runge-Kutta, predictor-corrector methods; finite difference methods for PDEs.

Probability and Statistics: Probability axioms; conditional probability; Bayes' theorem; random variables: discrete and continuous; distributions: Bernoulli, Binomial, Poisson, Geometric, Uniform, Normal, Exponential, Gamma, Beta; moment generating functions and characteristic functions; law of large numbers; central limit theorem; joint distributions, marginal and conditional distributions; estimation: method of moments, MLE; confidence intervals; hypothesis testing: type I and type II errors, t-test, chi-square test, F-test; regression and correlation; non-parametric tests.

Discrete Mathematics: Logic and proof techniques: propositional and predicate logic, proof by induction, contradiction; set theory; relations and functions; combinatorics: permutations, combinations, pigeonhole principle, inclusion-exclusion; recurrence relations and generating functions; graph theory: paths, cycles, connectivity, Euler and Hamilton circuits, planar graphs, colouring, matching; trees: spanning trees, Kruskal's and Prim's algorithms; Boolean algebra and switching theory; automata theory: DFA, NFA, regular languages, context-free grammars.

Operations Research: Linear programming: graphical method, simplex method, big-M and two-phase methods, duality theory, sensitivity analysis; transportation and assignment problems; network flow: shortest path, maximum flow, min-cut theorem; integer programming: branch and bound, cutting plane; dynamic programming: principle of optimality, shortest path, knapsack; game theory: two-person zero-sum, saddle point, mixed strategies; queuing theory: M/M/1, M/M/c models; simulation; inventory models: EOQ with and without shortage.